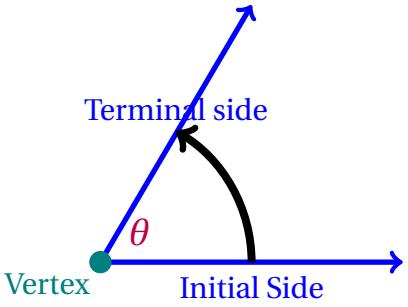
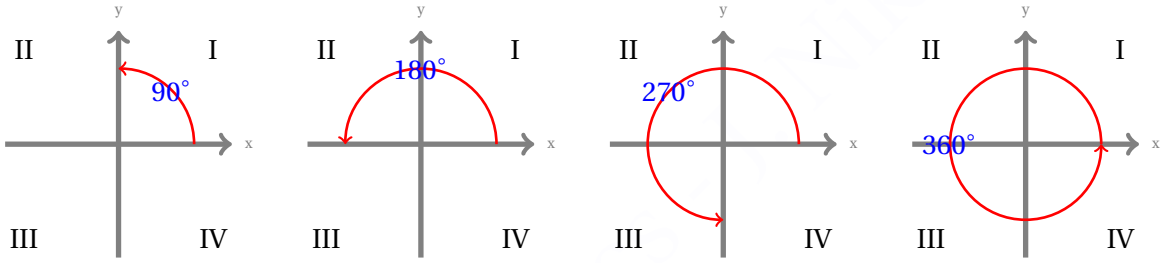
5.1: Angles

- **An Angle**



- **The quadrants:**



- In **converting** radians to degrees or vice versa, use the following formula:

$$\frac{\text{Radians}}{\pi} = \frac{\text{Degrees}}{180^\circ}$$

For example, 45° coverts to $\frac{\pi}{4}$ rad, 30° coverts to $\frac{\pi}{6}$ rad, and 60° coverts to $\frac{\pi}{3}$.

Coterminal Angles:

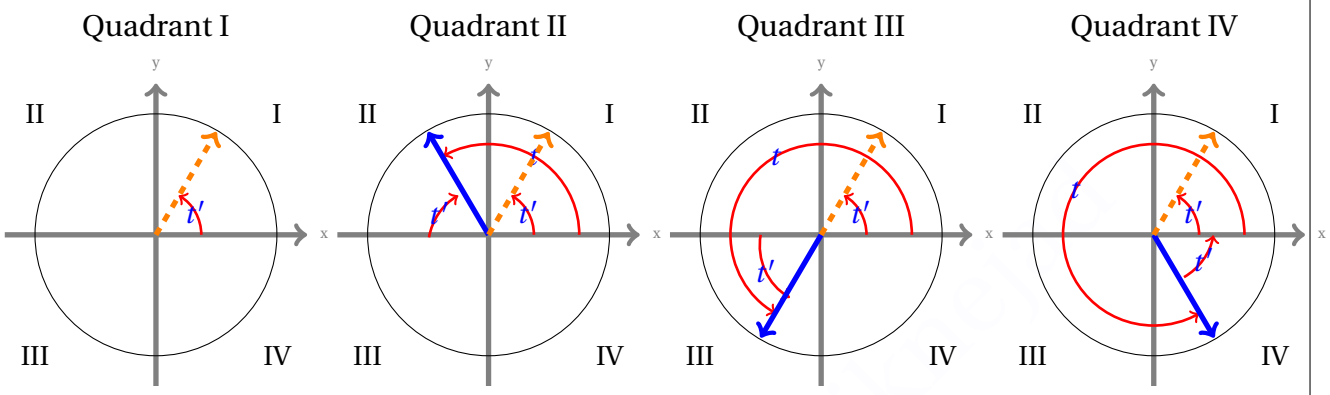
Coterminal angles are two angles in standard position that have the same terminal side.

- How to find the smallest positive coterminal angle for angle θ, **Principal angle**,

1. If $\theta > 2\pi$ subtract 2π repeatedly until the angle you get as a result is a positive angle less than 2π.
2. If $\theta < 0$ add 2π repeatedly until the angle you get as a result is a positive angle less than 2π.

For example, 45° and 405° are coterminal angles; equivalently, $\frac{\pi}{4}$ rad and $\frac{9\pi}{4}$ are coterminal angles; also, $\frac{\pi}{4}$ and $\frac{17\pi}{4}$ are coterminal angles.

Reference Angles



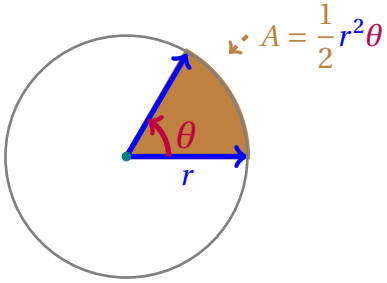
The reference angle of an angle t is the size of the acute angle t' that is made by the terminal side of t and x-axis.

For example, $\frac{\pi}{4}$ is the reference angle for $\frac{3\pi}{4}$ rad, $\frac{5\pi}{4}$ rad, and $\frac{7\pi}{4}$ rad.

Area of a Sector

The area of a sector of a circle with radius r subtended by an angle θ, measured in radians, is

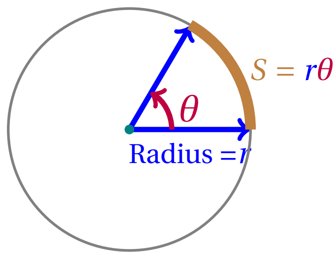
$$A = \frac{1}{2}r^2\theta$$



Arc Length and Sector Area

- **Arc length in a Circle** In a circle of radius r, the length of an arc subtended by an angle with measure θ in radians is

$$S = r\theta.$$



Angular Speed

- As a point moves along a circle of radius r, its angular speed, ω, is the angular rotation θ, in radians, per unit time, t.

$$\omega = \frac{\theta}{t}$$

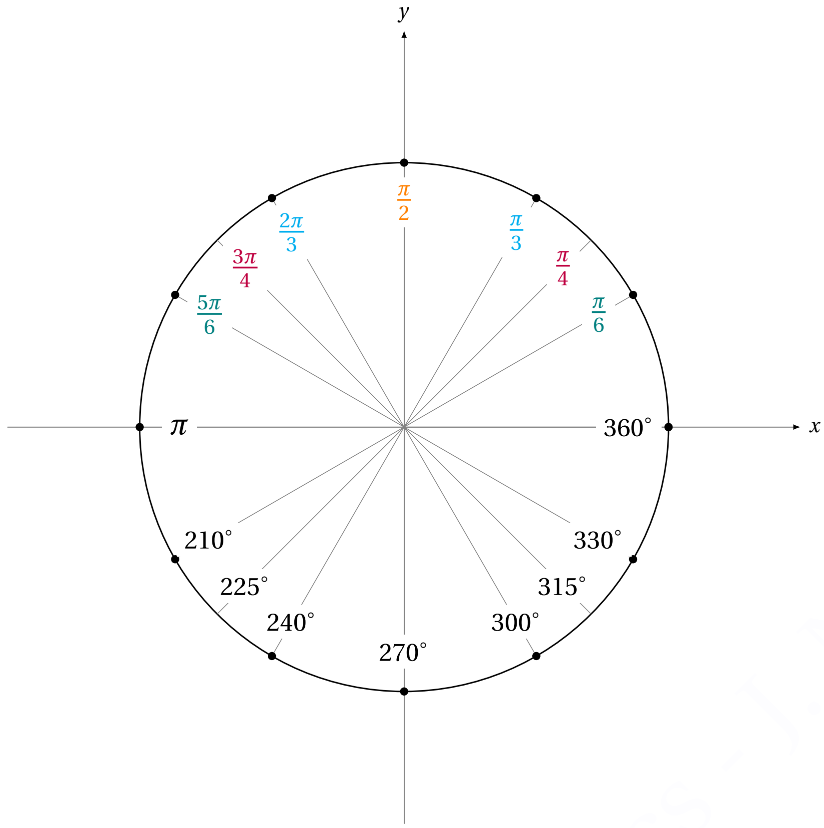
The linear speed, v, of the point can be found as the distance traveled, arc length, per unit time, t

$$v = \frac{s}{t} = \frac{\text{Total Arc length}}{\text{time}}$$

When the angular speed is measured in radians per unit time, linear speed and angular speed are related by the equation

$$v = r\omega$$

1. For each angle in radians, find the measure in degrees. For each angle in degrees find the angle in radians.



2. The measures of two angles in standard position are $\frac{5\pi}{6}$ and $\frac{29\pi}{6}$. Determine whether the angles are coterminal.

3. Find two positive angles and two negative angles that are coterminal with $\frac{5\pi}{3}$.

4. Find the smallest positive **coterminal angle** for each of the following angles.

Angle	$\frac{10\pi}{3}$	$\frac{8\pi}{3}$	$\frac{11\pi}{4}$	$\frac{11\pi}{3}$	$\frac{13\pi}{6}$	$\frac{-\pi}{3}$	$\frac{-5\pi}{6}$	$\frac{-5\pi}{3}$	$\frac{-3\pi}{4}$
Coterminal									

5. (A) The area of a circle is 98 cm^2. Find the area of a sector of this circle that subtends a central angle of $\pi/4$ rad.

   (B) A sector of a circle of radius 28 miles has an area of 784 miles^2. Find the central angle of the sector.

6. (A) Find the length of an arc that subtends a central angle of 45° in a circle of radius 10 m.

   (B) A central angle θ in a circle of radius 2m is subtended by an arc of length 2.5 m. Find the measure of θ in degrees and in radians.

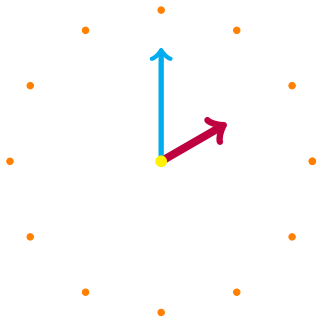
7. A car travels 6 km. Each of its tires makes 2640 revolutions. What is the radius of a tire in meters?

8. The diameter of wheels of a car is 75 cm and they are revolving at angular speed of 1000 radian per minute. What is the linear velocity of the car in km/hr?

9. (a) What is the angular speed of minute hand?

   (b) What is the angular speed of hour hand?

   (c) After 2 o'clock, at what time do hour hand and minute hand meet first?



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